163A-class Math Review Notes

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1 Introduction

This note is for the math review in 163A-Quantum Mechanics and Basic Spectroscopy class.

2 Complex numbers

$i^2 = -1$, $i = \sqrt{-1}$

$z = x + iy$, $x = Re(z)$, $y = Im(z)$

$z.z^* = (x + iy).(x - iy) = x^2 - ixy + ixy + y^2 = x^2 + y^2$

complex number z can also be expressed with r and θ:

$z = re^{i\theta}$, $r = \sqrt{x^2 + y^2}$, $tan\theta = \frac{y}{x}$

Euler equation:
$e^{i\theta} = cos\theta + isin\theta$

$z.z^* = (re^{i\theta}).(re^{-i\theta}) = r^2 = x^2 + y^2$

3 Derivative

. **Product rule**:
if $h(x) = f(x)g(x)$, then $h'(x) = f'(x)g(x) + f(x)g'(x)$

. **Chain rule**:
if $h(x) = f(g(x))$ then $h'(x) = f'(g(x)).g'(x)$

4 Integration

Review single and multi-variable integration, spherical coordinate integration

One important rule - Integration by parts:
$\int udv = uv - \int vdu$

5 Differential Equations

Ordinary differential equation: only one independent variables such as x:

$\frac{d^2X(x)}{dx^2} - kX(x) = 0$

Partial differential equation: more than one independent variable

$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u(x,t)}{\partial t^2}$ (wave equation)

$i\hbar\frac{\partial \phi(x,t)}{\partial t} = H\phi(x,t) = (\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x))\phi(x,t)$ time-dependent Schrodinger Equation

Linear differential equation:
$A(x)y^{(n)} + by^{(n-1)} + cy = g(x)$

"linear": A, g function of x only, y and its derivatives appear to be first order
(n): order of linear ODE (ordinary differential equation)
if g(x)=0, homogeneous ODE, otherwise non-homogeneous ODE

General form for second order linear homogeneous ODE:
$y'' + P(x)y' + Q(x)y = 0$

$y = C_1y_1 + C_2y_2$ (for second order ODE, general solution is a linear combination of two solutions, C_1 and C_2 are two constants to be determined by the boundary condition)

A tentative guess of solution form for y_1 and y_2 is $e^{\alpha x}$
$y'' + Py' + Qy = 0$ (1)
$\alpha^2 e^{\alpha x} + P\alpha^2 e^{\alpha x} + Qe^{\alpha x} = 0$
$(\alpha^2 + P\alpha^2 + Q)e^{\alpha x} = 0$
$e^{\alpha x}$ can not be zero which gives a zero solution, so $\alpha^2 + P\alpha^2 + Q = 0$
from which we obtain the two roots of α: α_1, α_2

In general, the solution of Eq. (1) can be written as:
$y = C_1e^{\alpha_1 x} + C_2e^{\alpha_2 x}$

6 Matrices

a. Vector

a column vector: $A = \begin{pmatrix} \text{a} \\ \text{b} \\ \text{c} \end{pmatrix}$;

a line vector: $B = \begin{pmatrix} \text{d e f} \end{pmatrix}$

b. Outer product

$$A.B = \begin{pmatrix} \text{a} \\ \text{b} \\ \text{c} \end{pmatrix} \begin{pmatrix} \text{d e f} \end{pmatrix} = \begin{Bmatrix} \text{ad ae af} \\ \text{bd be bf} \\ \text{cd ce cf} \end{Bmatrix}$$

c. Inner product

$$B.A = \begin{pmatrix} \text{d e f} \end{pmatrix} \begin{pmatrix} \text{a} \\ \text{b} \\ \text{c} \end{pmatrix} = da + eb + fc$$

d.Vector summation

$$A = \begin{pmatrix} \text{a} \\ \text{b} \\ \text{c} \end{pmatrix}; B = \begin{pmatrix} \text{d} \\ \text{e} \\ \text{f} \end{pmatrix}; A + B = \begin{pmatrix} \text{a+d} \\ \text{b+e} \\ \text{c+f} \end{pmatrix};$$

e. Determinant

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & k \end{vmatrix} = a \begin{vmatrix} e & f \\ h & k \end{vmatrix} - b \begin{vmatrix} d & f \\ g & k \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$\begin{vmatrix} d & e \\ g & h \end{vmatrix} = dh - eg$$

e. Matrix transpose

$$A = \begin{Bmatrix} \text{a b} \\ \text{c d} \end{Bmatrix}; A^T = \begin{Bmatrix} \text{a c} \\ \text{b d} \end{Bmatrix};$$

f. Conjugate of matrix

$$A = \begin{Bmatrix} \text{a b+i} \\ \text{c+i d} \end{Bmatrix}; \bar{A} = \begin{Bmatrix} \text{a b-i} \\ \text{c-i d} \end{Bmatrix};$$

g. Conjugate transpose

$$A = \begin{Bmatrix} \text{a b+i} \\ \text{c+i d} \end{Bmatrix}; A^+ = \begin{Bmatrix} \text{a c-i} \\ \text{b-i d} \end{Bmatrix};$$

h. Hermitian matrix

if $A = A^+$, then A is a Hermitian matrix which can only have real eigenvalues, used to represent quantum mechanical operator

i. Eigenvalue of matrix

$\hat{M}|\phi\rangle = E|\phi\rangle$

$|\phi\rangle$ is eigenvector, E is eigenvalue.
if $\hat{M}$ is Hermitian, E can be only real.
For quantum mechanics, time independent Schrodinger equation: $\hat{H}|\phi\rangle = E|\phi\rangle$